

SIMP from a strong U(1) gauge theory with a monopole condensation

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We provide a variant model of strongly interacting massive particle (SIMP), where composite dark matter comes from a strongly interacting U(1) theory. We first explain a non-Abelian version of the model with an additional singlet field, which is mixed with the Higgs field to maintain the kinetic equilibrium between the hidden and Standard Model (SM) sectors. The mixing leads to signals that would be detected by future collider experiments, direct DM detection experiments, and beam-dump experiments. Then we investigate a U(1) theory with a scalar monopole, where U(1) charged particles are confined by monopole condensation. In this model, the radial component of monopole can mix with the Higgs field, so that we do not need to introduce the additional singlet field.

I. INTRODUCTION

Strongly interacting confining theories are widely considered as a physics beyond the Standard Model (SM). In the literature, they focused on non-Abelian gauge theories to realize strong interactions and confinement because they are asymptotic free and become strong at low energy. However, Abelian gauge theories can realize confinement by a monopole condensation, which was discussed to aim to understand the confinement of QCD [1]. There are many theoretical studies to reveal properties of theories with electrons and monopoles in $\mathcal{N} = 2$ [2, 3] and $\mathcal{N} = 1$ supersymmetry [4–7]. In cosmology, confining U(1) theories have some advantage compared with non-Abelian models. For example, a particle content is economical [8] and there is no baryon state at low energy [9].

In the literature, a strongly interacting massive particle (SIMP) is widely considered as a candidate for dark matter (DM) because its self-interaction can address some astrophysical offset such as “core v.s. cusp” problem and “too-big-to-fail” problem [10–13]. A recent observation of Abell3827 cluster also favours SIMP DM models [14] (see also Ref. [15]). It has been pointed out that $\mathcal{O}(100)$ MeV DM can have a observed relic density by the freezeout mechanism via $3 \rightarrow 2$ annihilation process and have a correct scattering cross section required to explain the above astrophysical problems and observations [16]. In particular, such a model can be realized by a low energy effective theory of a strongly interacting non-Abelian gauge theory [17]. The SIMP models require that the DM sector is in kinetic equilibrium with the SM sector so that the DM does not become hot via $3 \rightarrow 2$ annihilation process. In order to realize the kinetic equilibrium, they may introduce a hidden U(1) gauge boson which has a small kinetic mixing with SM U(1)_Y gauge boson [18, 19].

In this paper, we provide simple models to realize the SIMP mechanism that predicts correct relic DM abundance and scattering cross section indicated by astrophysical observations. First we consider a non-Abelian gauge theory with a singlet field in the hidden sector.

Assuming a mixing between the singlet field and the SM Higgs field, we can maintain the kinetic equilibrium between the hidden and SM sectors. The mixing effect leads to signals for future collider experiments, direct DM detection experiments, and beam-dump experiments. Then we provide a SIMP model in an Abelian gauge theory that is confined due to a monopole condensation. In this model, the radial component of monopole plays the role of the singlet field of the former model, which naturally realize the kinetic equilibrium between the hidden and SM sectors. It is outstanding that the monopole plays the roles of U(1) confinement and mediator between the hidden and SM sectors.

II. SIMP WITH A SINGLET FIELD

A. SIMP and its thermal relic

First, we consider a variant SIMP model in a non-Abelian gauge theory. We introduce a singlet field S with N_F pairs of hidden quarks Q_i and \bar{Q}_i , which are charged under a hidden SU(N) gauge symmetry in the fundamental and anti-fundamental representation, respectively. The hidden quarks interact with the singlet field via the following Yukawa interaction:

$$\mathcal{L}_{\text{int}} = \lambda S Q_i \bar{Q}_i + \text{h.c.}, \quad (1)$$

where we assume SU(N_F)_V flavour symmetry (see Table I). As we discuss in Sec. II C, we consider the case that the singlet field has a nonzero vacuum expectation value (VEV), which gives a mass for hidden quarks such as $m_Q = \lambda \langle S \rangle$. We also assume that there is a mixing between the singlet field S and the SM Higgs field h with a mixing parameter θ , which gives interactions between the hidden and SM sectors.

We assume that the SU(N) gauge interaction becomes strong at low energy and the hidden-quarks are confined at the low-energy scale. This implies that the chiral symmetry is broken at the confinement scale, so that the low-energy effective theory can be described by pions π_i

	$SU(N_F)_V$	$SU(N)$
Q_i	\square	\square
\bar{Q}_i	$\bar{\square}$	$\bar{\square}$

TABLE I. Charge assignment for hidden matter fields in a model considered in Sec. II.

($i = 1, 2, 3, \dots, N_F - 1$) and baryons in the hidden sector. The mass of pions m_π may be roughly given by

$$m_\pi \sim \sqrt{m_Q \Lambda}, \quad (2)$$

where Λ is the dynamical scale of $SU(N)$. The hidden-pion decay constant f_π is naively related to the dynamical scale of $SU(N)$ such as $4\pi f_\pi / \sqrt{N} \approx \Lambda$. Note that the pions are stable due to the $SU(N_F)_V$ flavour symmetry.

Once we omit the dynamics of the singlet field, our model is equivalent to the one discussed in Ref. [17]. They have found that the thermal relic density of pions can be consistent with the observed DM density and their self-interaction cross section can address the tension between astrophysical observations and Λ CDM model when the pion mass m_π is about 100 – 500 MeV and the pion decay constant f_π is about $m_\pi / (5 - 10)$. Although their analysis is based on the chiral perturbation theory, the expansion parameter $m_\pi \sqrt{N} / (4\pi f_\pi)$ is of order unity and the perturbation may break down. In fact, it has been discussed that the next-to-leading order effect becomes relevant in the interesting parameter region, so that there are $\mathcal{O}(1)$ uncertainties in their analysis [20]. In this paper, we assume $m_\pi \sim 4\pi f_\pi / \sqrt{N} \simeq \Lambda$ and use the naive dimensional analysis to estimate properties of pions, which we identify as self-interacting DM.

For the low energy effective theory, we write the effective lagrangian of pion fields by the naive dimensional analysis such as

$$\begin{aligned} \mathcal{L}_\pi = & -\frac{1}{2} \text{Tr} [\partial_\mu \pi \partial^\mu \pi] - \frac{m_\pi^2}{2} \text{Tr} [\pi \pi] + \mathcal{L}_{2 \rightarrow 2} \\ & + c'_{\text{WZW}} \frac{(4\pi)^3}{N^{3/2} \Lambda^5} \epsilon^{\mu\nu\rho\sigma} \text{Tr} [\pi \partial_\mu \pi \partial_\nu \pi \partial_\rho \pi \partial_\sigma \pi] + \dots, \end{aligned} \quad (3)$$

where c'_{WZW} is an $\mathcal{O}(1)$ parameter, $\mathcal{L}_{2 \rightarrow 2}$ represents terms that contribute to $\pi\pi \rightarrow \pi\pi$ scatterings, and the trace takes for the flavour indices. Terms with a odd number of pions are generically forbidden as long as CP-invariance is respected (we discuss the strong CP phase later). The forth term, however, respects the CP-invariance with a help of the anti-symmetric tensor $\epsilon^{\mu\nu\rho\sigma}$. When chiral symmetry is present, c_{WZW} has a quantization condition, but that is not essential here. Note that it vanishes unless the number of pions N_π is equal to or larger than five. Hereafter, we assume $N_F \geq 3$ ($N_\pi \geq 8$) and rewrite c'_{WZW} as $c_{\text{WZW}} \equiv (m_\pi / \Lambda)^5 c'_{\text{WZW}}$. We estimate the elastic scattering cross section for pions divided

by m_π such as

$$\begin{aligned} \frac{\sigma_{\text{ela}}}{m_\pi} &= \frac{(4\pi)^4 c_1^2}{8\pi N^2 m_\pi^3} \\ &\simeq 0.22 \text{ cm}^2/g \ c_1^2 N^{-2} \left(\frac{m_\pi}{1 \text{ GeV}} \right)^{-3}, \end{aligned} \quad (4)$$

where we use the naive dimensional analysis and c_1 is an $\mathcal{O}(1)$ parameter. Observations of cluster collisions, including the bullet cluster, implies $\sigma_{\text{ela}}/m_{\text{DM}} \lesssim 0.47 \text{ cm}^2/g$ and ellipticity on Milky way and cluster scales puts a constraint such as $\sigma_{\text{ela}}/m_{\text{DM}} \lesssim 0.5 \text{ cm}^2/g$ [21–25]. However, the cross section should be as large as this upper bound to address the "core-cusp" and "too-big-to-fail" problems [10–13]. Recently, it has been reported that there is an offset for the observation of Abell3827 cluster, which may be addressed by self-interacting DM with a cross section of $\sigma_{\text{ela}}/m_{\text{DM}} = 1.5 \text{ cm}^2/g$ [14]. These constraints and discussions have $\mathcal{O}(1)$ uncertainties due to, say, the difficulties of numerical simulations, so that in this paper we require that DM has self-interactions with the cross section of order $\sigma_{\text{ela}}/m_{\text{DM}} = 0.1 - 1 \text{ cm}^2/g$ to address the above astrophysical problems. Equation (4) then implies $m_\pi = \mathcal{O}(1) \text{ GeV}$, which is consistent with the original works within an $\mathcal{O}(1)$ uncertainty [17].

Suppose that the hidden sector is in kinetic equilibrium with the SM sector, which is justified in the next subsection. When we can neglect $2 \rightarrow 2$ annihilation of pions, their Boltzmann equation can be written as [16]

$$\dot{n}_\pi + 3Hn_\pi = - (n_\pi^3 - n_\pi^2 n_\pi^{\text{eq}}) \langle \sigma v^2 \rangle_{3 \rightarrow 2}. \quad (5)$$

Here, the thermal number density of π is given by

$$n_\pi^{\text{eq}}(T) \simeq N_\pi \left(\frac{m_\pi T}{2\pi} \right)^{3/2} e^{-m_\pi/T}, \quad (6)$$

and the thermally-averaged cross section of $3 \rightarrow 2$ scattering process is calculated as [17]

$$\langle \sigma v^2 \rangle_{3 \rightarrow 2} = \frac{(4\pi)^6 c_{\text{WZW}}^2 375 \sqrt{5}}{2\pi N_F N^3 m_\pi^5} \frac{T^2}{m_\pi^2}, \quad (7)$$

where T is a temperature of hidden sector and we assume $N_F \gg 1$ for simplicity. This implies that the pions freeze out at a temperature below its mass scale and the resulting relic density is given by

$$n_\pi^{(\text{FO})} \simeq \left(\frac{3H(T_F)}{\langle \sigma v^2 \rangle_{3 \rightarrow 2}} \right)^{1/2}, \quad (8)$$

where $H(T_F)$ is the Hubble parameter at the time of pion freeze-out. As a result, we obtain

$$\frac{\rho_\pi}{s} \simeq 0.2 \text{ eV} \ c_{\text{WZW}}^{-1} N_F^{1/2} N^{3/2} \left(\frac{m_\pi}{1 \text{ GeV}} \right)^{3/2}, \quad (9)$$

where T_F is the temperature at the pion freeze-out and we define $x_F \equiv m_\pi / T_F$ ($\simeq 24$). Thus, the pion abundance is consistent with the observed DM abundance of $\rho_{\text{DM}}^{(\text{obs})} / s \simeq 0.4 \text{ eV}$ when $m_\pi = \mathcal{O}(1) \text{ GeV}$. In this parameter region, the self-interaction cross section of Eq. (4) is consistent with the value indicated by astrophysical observations.

B. Kinetic equilibrium between two sectors

Now let us check that the kinetic equilibrium between the hidden and SM sector is fulfilled until the pions freeze out. First, note that the singlet field S interacts with the pions via a coupling of

$$\mathcal{L}_{\text{int}} = \frac{m_\pi^2}{2} \frac{S}{\langle S \rangle} \text{Tr} [\pi\pi], \quad (10)$$

where $\langle S \rangle$ is the VEV of S at the vacuum. Note that the apparent singularity at $\langle S \rangle \rightarrow 0$ is cancelled by m_π^2 [see Eqs. (1) and (2)]. Assuming a mixing between the singlet field S and the SM Higgs field h , we can realize the kinetic equilibrium between the hidden and SM sectors. We denote the mixing parameter as θ .

First, we consider the case of $r \equiv m_S/m_\pi \gg 1$, where m_S is singlet mass. In this case, we can integrate out the singlet field to consider scatterings between pions and SM particles. Then we obtain the four-point interaction between the pions and electrons or muons by the mixing effect and the SM Yukawa interactions.¹ Here we explain the contribution of scatterings with muons because they dominate those with electrons for the case of $m_\pi \gtrsim 0.2$ GeV. The cross section is roughly given by

$$\sigma_{\pi\mu \rightarrow \pi\mu} \approx \frac{\theta^2 y_\mu^2}{8\pi} \frac{m_\pi^2 m_\mu^2}{\langle S \rangle^2 m_S^4}, \quad (11)$$

where m_μ ($\gtrsim T_F$) is the muon mass and y_μ is the muon Yukawa coupling. In order to be thermalized between successive $3 \rightarrow 2$ scatterings, pions need to lose kinetic energy about m_π obtained from $3 \rightarrow 2$ scattering. Thus we estimate $\langle \sigma v E_{\text{ex}}/m_\pi \rangle_{\pi\mu \rightarrow \pi\mu} \approx \sigma_{\pi\mu \rightarrow \pi\mu} m_\mu/m_\pi$ where E_{ex} ($\approx m_\mu$) represents exchanged energy. In order to maintain the kinetic equilibrium between the two sectors at the pion freeze-out, we need to satisfy

$$\left\langle \sigma v \frac{E_{\text{ex}}}{m_\pi} \right\rangle_{\pi\mu \rightarrow \pi\mu} n_\mu^{\text{eq}}(T_F) \gtrsim \langle \sigma v^2 \rangle_{3 \rightarrow 2} (n_\pi^{\text{eq}}(T_F))^2 \simeq H(T_F). \quad (12)$$

Here, the thermal number density of muons is given by

$$n_\mu^{\text{eq}}(T) \simeq 4 \left(\frac{m_\mu T}{2\pi} \right)^{3/2} e^{-m_\mu/T}. \quad (13)$$

Thus we have a constraint on θ depending on m_π and m_S . We find that it could not be consistent with present constraints (discussed below) for the case of $r \gg 1$.

Next, we consider the case that m_S is larger than but close to m_π ($r \approx 1$). In this case, the number density of S in the thermal plasma cannot be neglected at the

time of pion freeze-out though it is suppressed by the Boltzmann factor. The singlet field S elastically interacts with the pions with the cross section of order $\sigma_{S\pi \rightarrow S\pi} \approx m_\pi^2/(8\pi \langle S \rangle^4)$, where we use $m_S \simeq m_\pi$. The exchanged energy E_{ex} is now roughly given by m_π . Therefore, these particles are in kinetic equilibrium when

$$\left\langle \sigma v \frac{E_{\text{ex}}}{m_\pi} \right\rangle_{S\pi \rightarrow S\pi} n_S^{\text{eq}}(T_F) \gtrsim \langle \sigma v^2 \rangle_{3 \rightarrow 2} (n_\pi^{\text{eq}}(T_F))^2 \simeq H(T_F) \quad (14)$$

$$\leftrightarrow r \equiv \frac{m_S}{m_\pi} \leq r_{\text{max}}, \quad (15)$$

$$r_{\text{max}} \simeq 1 + \frac{1}{x_F} \ln \left[\frac{\rho_\pi}{s} \frac{s(T_F)}{m_\pi H(T_F)} \frac{\lambda^4}{8\pi m_\pi^2} \frac{1}{N_\pi} \left(\frac{m_S}{m_\pi} \right)^{3/2} \right] \\ \simeq 1 + \frac{1}{x_F} \ln \left[7.2 \times 10^6 N_\pi^{-1} r_{\text{max}}^{3/2} \lambda^4 \left(\frac{m_\pi}{1 \text{ GeV}} \right)^{-2} \right], \quad (16)$$

where $n_S^{\text{eq}}(T_F)$ is the thermal number density of S at $T = T_F$:

$$n_S^{\text{eq}}(T_F) \simeq \left(\frac{m_S T_F}{2\pi} \right)^{3/2} e^{-m_S/T_F}. \quad (17)$$

Here we implicitly assume that S is kinetically equilibrated with the SM sector via the decay and inverse-decay processes between successive $S\pi \rightarrow S\pi$ elastic scatterings. In fact, S and π are in kinetic equilibrium with the SM sector when Eq. (14) is satisfied and the decay and inverse decay rate of S is larger than $H(T_F) n_\pi^{\text{eq}}/n_S^{\text{eq}}$.² Since the decay rate of S is proportional to θ^2 , we rewrite the latter condition as

$$\theta \gtrsim \sqrt{\frac{8\pi H(T_F)}{y_\mu^2 m_S} N_\pi \left(\frac{m_\pi}{m_S} \right)^{3/2} e^{-m_\pi/T_F + m_S/T_F}} \\ \simeq 2.3 \times 10^{-7} N_\pi^{1/2} r^{-5/4} e^{x_F(r-1)/2} \left(\frac{m_\pi}{1 \text{ GeV}} \right)^{1/2} \quad (18)$$

Thus, we have an upper bound on r (r_{max}) and a lower bound on θ to realize kinetic equilibrium between two

² This condition comes from the following discussion, for example. Suppose that the decay and inverse-decay of S occur more rapidly than the $S\pi \rightarrow S\pi$ elastic scattering process (which rate is given by $\langle \sigma v E_{\text{ex}}/m_\pi \rangle_{S\pi \rightarrow S\pi} n_\pi^{\text{eq}}$). In this case, where the above condition follows from Eq. (14), the kinetic equilibrium is reached between π and S . In the other case, i.e., if $S\pi \rightarrow S\pi$ elastic scattering process occurs more rapidly than the decay and inverse-decay of S , the energy of each π is reduced by a factor of $n_\pi^{\text{eq}}/(n_\pi^{\text{eq}} + n_S^{\text{eq}})$ and each S obtains the energy of $E_\pi n_\pi^{\text{eq}}/(n_\pi^{\text{eq}} + n_S^{\text{eq}})$. Here, E_π is the initial energy of π after every $3 \rightarrow 2$ process and is of order m_π , and n_π^{eq} is thermal number densities of π . Then, the energy density of S is transferred to the SM plasma by its decay and inverse-decay processes. In every decay and inverse-decay process, the total energy density of S and π is reduced by a factor of $n_S E_\pi n_\pi^{\text{eq}}/(n_\pi^{\text{eq}} + n_S^{\text{eq}})$. Therefore, in order to reduce the initial energy of π by of order E_π between successive $3 \rightarrow 2$ scatterings, the decay and inverse-decay rate should occur $(n_\pi^{\text{eq}} + n_S^{\text{eq}})/n_S^{\text{eq}}$ ($\simeq n_\pi^{\text{eq}}/n_S^{\text{eq}}$) times. Finally we obtain the condition used in the main part of this paper.

¹ The SM pions and photons can interact with the hidden pions and may contribute to their kinetic thermalization. We neglect it for simplicity because their contributions are the same order with that of muons.

sectors. The lower bounds of Eq. (18) are plotted as blue curves in Fig. 1 for $r = 1.3, 1.5, 1.7$ ($r_{\max} \simeq 1.7$) and $\lambda = 1$ and for $r = 1.5$ ($r_{\max} \simeq 2.1$) and $\lambda = 4\pi$.

Here we should check that the annihilation of pions into singlet fields is inefficient at $T = T_F$ to justify the Boltzmann equation of Eq. (5). The annihilation rate is suppressed by a factor of $\exp[-x_F(r-1)]$ compared with the left-hand side of Eq. (14). We find that the annihilation is inefficient when $r \gtrsim 1.1, 1.3, 1.5$ for $\lambda = 0.1, 1, 4\pi$, respectively. Therefore, when $1.1 \lesssim r \lesssim 1.3$ for $\lambda = 0.1$, $1.3 \lesssim r \lesssim 1.7$ for $\lambda = 1$, and $1.5 \lesssim r \lesssim 2.1$ for $\lambda = 4\pi$, the pions are in kinetic equilibrium with the SM sector and their annihilation can be neglected. For each λ , we have the lower bound on r , so that we cannot realize kinetic equilibrium in the lower-shaded region in Fig. 1.

The nonzero mixing between the singlet field and the SM Higgs is constrained by the measurements of Higgs decay width at the LHC. The result is given by $\theta \lesssim 5.2 \times 10^{-2}/(N_F N \lambda)$ [26–28], which is shown as the horizontal green line in Fig. 1 for the case of $N_F = 3$, $N = 2$, $r = 1.5$, and $\lambda = 1$ and 4π . Another constraint comes from B decays at LHCb for $m_S \lesssim 5$ GeV. The mixing parameter should be $\theta \lesssim (5-30) \times 10^{-4}$ for $m_S = [0.3, 5]$ GeV though the regions $m_S = 2.95-3.18$ GeV and $m_S = 3.59-3.77$ GeV are vetoed in the experimental searches [29]. It is plotted as the red curve in Fig. 1 for the case of $r = 1.5$. Our predictions of mixing parameter for $r \lesssim 1.7$ are below the present upper bounds.

International linear collider (ILC) experiment as well as LHC would precisely measure Higgs decay width, so that we would find a nonzero mixing between the hidden singlet field and the SM Higgs field. The mixing effect also leads to DM direct detection signals (see, e.g., Ref. [28]). The present upper bound on the mixing parameter is larger than the above collider experiments [30], while some allowed region in Fig. 1 will be searched by future DM direct detection experiments, such as NEWS [31] and Super-CDMS SNOLAB [32]. Beam-dump experiments can search S with a small mixing parameter. The CHARM collaboration puts a constraint for $m_\pi \lesssim 0.36$ GeV/ r [33, 34], which is shown as magenta curves in the figures. We also plot future sensitivity of beam-dump experiment by the SHiP facility as magenta-dashed curve [35]. The regions inside the magenta and magenta-dashed curves are excluded and will be probed by these beam-dump experiments, respectively. From Fig. 1, we can see that a large parameter region will be searched in the near future.

Finally, let us comment on a strong CP phase in the hidden sector. The CP violating phase θ_{CP} of $SU(N)$ interaction gives a term like $(4\pi\theta_{CP}m_\pi)/\sqrt{N}\text{Tr}[\pi\pi\pi]$ in the low energy. It leads to unwanted $\pi + \pi \rightarrow \pi + S$ process, so that we have an upper bound on θ_{CP} such as $\theta_{CP} \lesssim 0.006N^{1/2}/\lambda$ for $r = 1.3$, $\theta_{CP} \lesssim 0.06N^{1/2}/\lambda$ for $r = 1.5$, and $\theta_{CP} \lesssim 0.6N^{1/2}/\lambda$ for $r = 1.7$. We can explain such a small θ_{CP} by forbidding the term by CP symmetry. Or, we can realize the Peccei-Quinn (PQ)

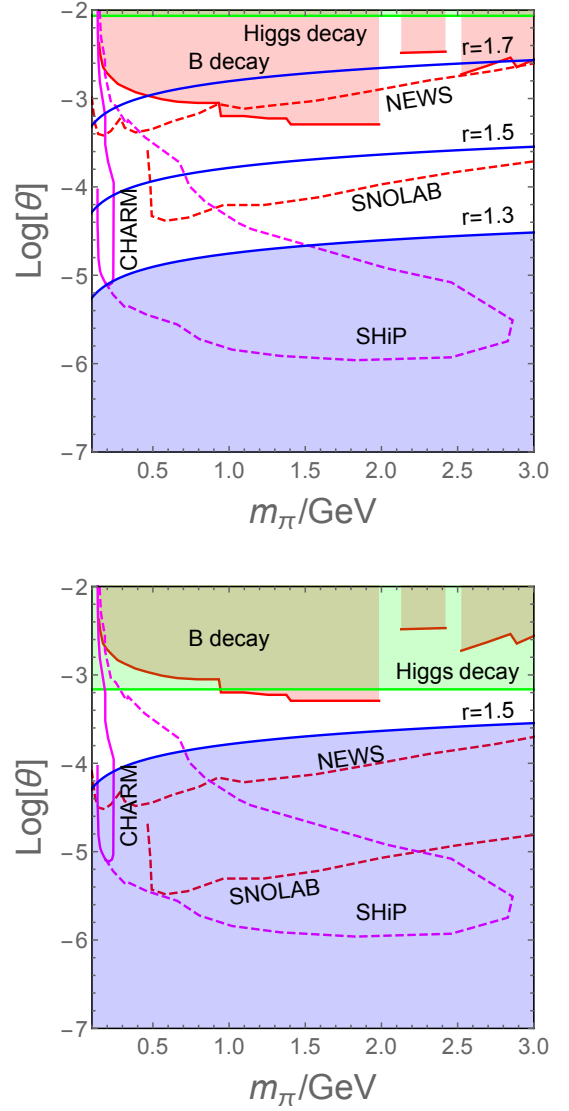


FIG. 1. Condition for kinetic equilibrium between the hidden and SM sectors for the case of $r \approx 1$. Above the blue curve for each value of r , we can maintain kinetic equilibrium. The green and red curves are the present upper bound on θ by the measurements of Higgs decay width [26–28] and B decay [29], respectively. The red-dashed curves are future sensitivities of DM direct detection experiments [31, 32]. The region inside magenta curve is excluded by a beam-dump experiment [33, 34], while that inside magenta-dashed curve is future sensitivity [35]. To plot the constraints and sensitivities, we take $r = 1.5$ and $\lambda = 1$ (upper panel) and 4π (lower panel).

mechanism in the hidden sector when we replace the singlet field S with a complex scalar field with a global PQ symmetry [36, 37]. After the PQ symmetry is spontaneously broken by the VEV of S , the CP violating phase is cancelled by the VEV of its phase component called axion. Note that when the axion mass is larger than the pion mass, we can neglect its effect on our analysis.

	$SU(N_F)_V$	$U(1)_H$	$U(1)_Y$
ψ_i	\square	1	0
$\bar{\psi}_i$	$\bar{\square}$	-1	0

TABLE II. Charge assignment for hidden matter fields in a model considered in Sec. III A.

C. Singlet potential

Here we explicitly write a model with a nonzero mixing between S and h as an example. We may write the potential of the singlet field and the SM Higgs field such as³

$$V(S, H) = A_S S + \frac{1}{2} m_S^2 S^2 + \frac{1}{3} B_S S^3 + \frac{1}{4} \lambda_S S^4 + m' S |H|^2 + \frac{g}{2} S^2 |H|^2 + V(|H|^2), \quad (19)$$

where m_S , m' , and B_S are parameters with mass-dimension one, g and λ_S are dimensionless parameters, A_S is a parameter with mass-dimension three, and $V(|H|^2)$ is the Higgs potential. The singlet field S acquires a nonzero VEV due to the first and the third terms. Note that we need $\langle S \rangle = m_Q/\lambda$. After H obtains a VEV at the electro-weak phase transition, the fifth and sixth terms lead to a mixing between S and the SM Higgs field such as $\theta \simeq (m' v_h + g \langle S \rangle v_h)/(2m_h^2)$, where v_h ($\simeq 246$ GeV) and m_h ($\simeq 125$ GeV) are the Higgs VEV and mass, respectively. We can obtain a small but nonzero mixing that is consistent with Eq. (18).

III. MODEL WITH STRONG $U(1)$

In this section, we consider another SIMP model in a strongly-interacting Abelian gauge theory. We consider a hidden Abelian gauge theory with a scalar monopole ϕ and N_F pairs of hidden electrons and positrons ψ_i and $\bar{\psi}_i$. The charge assignment for the hidden electrons and positrons are shown in Table II. We denote the electric coupling as g_e and the magnetic coupling as g_m , which satisfy Dirac quantization condition: $g_e g_m = 2\pi n$ ($n = 1, 2, 3, \dots$).

As we see below, the monopole develops condensation at low-energy scale to confine the $U(1)_H$ gauge interaction. Its radial component has a mass of order the confinement scale and can mix with the Higgs field. In this model, therefore, we do not need to introduce the singlet field to mediate two sectors.

A. Monopole condensation and mixing with Higgs

We write the potential of scalar monopole such as

$$V(\phi) = -\mu^2 |\phi|^2 + \lambda_\phi |\phi|^4, \quad (20)$$

where λ_ϕ is expected to be of order $(4\pi)^2$ by the naive dimensional analysis.⁴ At the minimum of the potential, the monopole develops a condensation such as $\sqrt{2} \langle |\phi| \rangle = \mu/\sqrt{\lambda_\phi} (\equiv v)$. Then, the mass of radial component of monopole, which we denote as φ ($\equiv \sqrt{2} |\phi|$), is given by $m_\varphi = \sqrt{2}\mu$. We expect that the mass of hidden $U(1)_H$ gauge boson m_v is of order m_φ . Hereafter, we assume m_v to be larger than m_π and neglect its effect except for in Sec. III B. After the monopole acquires the VEV, hidden electrons are attached by strings via the Meissner effect and are confined by the tension of the string [1]. Its tension μ_s determines the dynamical scale and is given as

$$\mu_s = \frac{g_e^2 g_m^2}{8\pi} v^2 \log \left(\frac{m_\varphi^2}{m_v^2} + 1 \right), \quad (21)$$

which is almost independent of g_e and g_m due to the Dirac quantization condition.

We have composite particles π_i below the confinement scale. There is no baryon state at low energy because baryons cannot be neutral under $U(1)_H$. Although in the previous section we add a singlet field S to maintain the kinetic equilibrium between the hidden and SM sectors, we can realize it via the mixing between the monopole and the SM Higgs field without adding the singlet field. We do not assume chiral symmetry in the hidden sector, which implies that pions have a mass of order the dynamical scale $m_\pi \approx \Lambda$.⁵ Note that we assume $SU(N_F)_V$ flavour symmetry to make pions stable (see Table. II).

Below the confinement scale, we have an interaction between the radial component of monopole φ and pions such as

$$\mathcal{L} = c_\varphi \frac{m_\pi^2}{2} \frac{\varphi}{v} \text{Tr} [\pi \pi], \quad (22)$$

where c_φ is an $\mathcal{O}(1)$ constant. Hereafter we take $c_\varphi = 1$. We introduce the following term to obtain the mixing

³ We define the origin of S such that the mass of hidden quarks vanishes at $\langle S \rangle = 0$.

⁴ Strictly speaking, it is not possible to write a Lagrangian for the system of both electrons and monopoles, and our equations like Eq. (20) should be regarded as a schematic picture of what is going on rather than a precise equation. Naive dimensional analysis should be applied to physical quantities like masses of particles and scattering amplitudes rather than ill-defined “parameters in the Lagrangian”.

⁵ Or we can just write an electron mass term to make pions massive. If the electron mass m_ψ is smaller than Λ , we might have an approximate chiral symmetry that is expected to be dynamically broken by the electron confinement. In this case, we have $m_\pi \simeq \sqrt{m_\psi \Lambda}$.

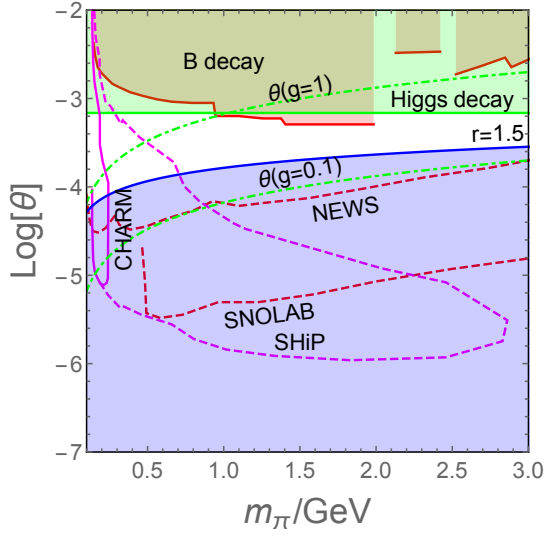


FIG. 2. Same as Fig. 1 but for the model considered in Sec. III A. The green dot-dashed curves represent the values of θ for $g = 0.1$ and 1 .

between φ and h :⁶

$$V_{\text{int}} = g |\phi|^2 |H|^2, \quad (23)$$

This term gives a mixing between φ and h with $\theta \simeq gv_h v / 2m_h^2$. Noting that $\lambda_\varphi \approx (4\pi)^2$, we find

$$\theta \simeq 4.4 \times 10^{-4} g \left(\frac{m_\varphi}{1 \text{ GeV}} \right). \quad (24)$$

Replacing m_S with m_φ and $\langle S \rangle$ with v , we can quote the calculations in Sec. II B. The result is shown in Fig. 2, where the hidden sector can be in kinetic equilibrium with the SM sector above the blue curve. The parameter θ is plotted as green dot-dashed curves, where we take $g = 0.1$ and 1 . Testabilities and constraints of this model are the same as the ones considered in the previous section (see the last two paragraphs in Sec. II B) except for an additional signal explained in the next subsection.

B. Kinetic mixing

Here we comment on a kinetic mixing between the $U(1)_H$ and $U(1)_Y$ gauge bosons [9]. First, note that the Abelian gauge theory may be conformal in the presence of monopole as well as electrons. In this case, its gauge field strength $F_{\mu\nu}$ has an scaling dimension larger than 2, which is guaranteed by the unitarity bound [38]. This

implies that the kinetic mixing term $\chi B_{\mu\nu} F^{\mu\nu}$ is an irrelevant operator and is suppressed at low energy. The resulting kinetic mixing parameter χ at low energy depends on the value of anomalous dimension of the $U(1)_H$ gauge boson $F^{\mu\nu}$, though we do not have information about it. Thus our model predicts a very small kinetic mixing between $U(1)_H$ and $U(1)_Y$ gauge bosons.

Once there is a mixing between $U(1)_H$ and $U(1)_Y$ gauge bosons, we may expect a term like⁷

$$\mathcal{L} \supset \frac{(4\pi)^2}{\Lambda^3} \chi \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} \text{Tr} [\pi \partial_\rho \pi \partial_\sigma \pi], \quad (25)$$

where we omit an $\mathcal{O}(1)$ uncertainty factor. This term leads to unwanted $\pi + \pi \rightarrow \pi + \gamma$ annihilation process, so that its cross section should be suppressed such as

$$\langle \sigma v \rangle_{\pi\pi \rightarrow \pi\gamma} n_\pi^{\text{eq}}(T_F) \lesssim \langle \sigma v^2 \rangle_{3 \rightarrow 2} (n_\pi^{\text{eq}}(T_F))^2 \simeq H(T_F), \quad (26)$$

where a rough estimation gives

$$\langle \sigma v \rangle_{\pi\pi \rightarrow \pi\gamma} \sim \chi^2 \frac{(4\pi)^4}{8\pi m_\pi^2} \left(\frac{T}{m_\pi} \right). \quad (27)$$

This can be rewritten as

$$\chi \lesssim 6 \times 10^{-5} \left(\frac{m_\pi}{1 \text{ GeV}} \right), \quad (28)$$

within an $\mathcal{O}(1)$ uncertainty. Such a small kinetic mixing parameter is consistent with our model because $\chi B_{\mu\nu} F^{\mu\nu}$ is an irrelevant operator and is suppressed at low energy as explained above.

The kinetic mixing is constrained by many experiments (see Refs. [18, 19]). A model-independent bound comes from electroweak precision tests because nonzero kinetic mixing modifies parameters in the EW sector. They put an upper bound such as $\chi \lesssim 2 \times 10^{-2}$ for $m_v \lesssim \mathcal{O}(1) \text{ GeV}$ [39]. In our model, the massive hidden photon dominantly decays into hidden pions for $m_v \gtrsim 2m_\pi$, in which case BaBar experiment puts a constraint as $\chi \lesssim 10^{-3}$ [40]. Even for $m_v \lesssim 2m_\pi$, it puts a similar constraint [41]. In the near future, Bell-II experiment can measure the mixing parameter of order 10^{-4} [39, 42]. We may not expect that we can observe kinetic mixing effect suppressed by Eq. (28).

⁶ The coupling constant g may be smaller than $\mathcal{O}(1)$ due to an anomalous dimension of monopole because our model may be conformal above the monopole and electron mass scale. Unfortunately, we cannot determine the anomalous dimension of the monopole.

⁷ In the presence of monopole, the gauge field strength does not satisfy the Bianchi identity. As a result, we have no reason that we can omit the term of Eq. (25). If $F_{\mu\nu}$ satisfies Bianchi identity and thus the gauge field is well-defined, in the mass eigenstate of gauge fields, only the Z-boson couples to $\epsilon^{\mu\nu\rho\sigma} \text{Tr} [\pi \partial_\rho \pi \partial_\sigma \pi]$ at the tree level. Then, Eq. (25) is expected to have a small coefficient (if exist) in Refs. [18, 19]. If Bianchi identity is not satisfied, operator mixing by the strong dynamics between $F^{\mu\nu}$ and $\epsilon^{\mu\nu\rho\sigma} \text{Tr} [\pi \partial_\rho \pi \partial_\sigma \pi]$ is not forbidden (or in other words, $F^{\mu\nu}$ can create/annihilate three pion states as well as one vector boson states) and Eq. (25) is generated by replacing $F^{\mu\nu}$ by $\epsilon^{\mu\nu\rho\sigma} \text{Tr} [\pi \partial_\rho \pi \partial_\sigma \pi]$ in $\chi B_{\mu\nu} F^{\mu\nu}$.

IV. DISCUSSION AND CONCLUSIONS

We have provided self-interacting DM models that explain the discrepancy between astrophysical observations and Λ CDM model. The models are based on low-energy effective theories of hidden QCD, where hidden pions are identified as SIMP DM. The thermal relic abundance of DM is determined by $3 \rightarrow 2$ scattering process and is consistent with the observed DM abundance as discussed in Ref. [17]. We have first investigated a non-Abelian gauge theory with a singlet field. The condition for kinetic equilibrium between the hidden and SM sectors can be realized by a mixing between the singlet field and the SM Higgs field. The nonzero mixing parameter will be measured by future experiments, such as LHC, ILC, NEWS [31], Super-CMDS SNOLAB [32], and SHiP [35].

Then we have provided a composite SIMP model originating from $U(1)_H$ confinement due to monopole condensation. In this case, the radial component of monopole can mix with the SM Higgs field, so that it mediate the hidden and SM sectors without introducing the additional singlet field. It is outstanding that the monopole

plays the roles of $U(1)_H$ confinement and mediator between two sectors. In addition, there is no unwanted baryon in this theory because baryons are charged under $U(1)_H$.

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